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M 004

MODEL PAPER

B.E. DEGREE EXAMINATION.

Fourth Semester

Electronics and Communication Engineering

MA 034 — RANDOM PROCESSES

Time : Three hours | Maximum : 100 marks

(Use of Statistical Tables allowed.)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Let X be a random variable taking values –1, 0 and 1 such that $P(X=-1) = 2P(X=0) = P(X=1)$. Find the mean of $2X-5$.

2. The moment generating function of a random variable X is given by $M_X(t) = e^{3(e^t-1)}$. Find $P(X=1)$.

3. A continuous random variable X has probability density function given by $f(x) = 3x^2;\ 0 \le x \le 1$. Find k such that $P(X > k) = 0.05$.

4. The joint probability density function of two random variables X and Y is given by $f_{XY}(x,y) = \frac{1}{8}x(x-y);\ 0 < x < 2;\ -x < y < x$ and otherwise. Find $f_{Y/X}(y/x)$.

5. Define Poisson random process. Is it a stationary process. Justify the answer.

6. Define (a) Wide sense stationary random process (b) ergodic random process.

7. A stationary random process $X = \{X(t)\}$ with mean 3 has auto–correlation function $R(z) = 16 + 9e^{-|\tau|}$. Find the standard deviation of the process.

8. Define cross–covariance function of two random processes and state two of its properties.

9. The auto–correlation function of a wide sense stationary random process is given by $R(\tau) = a^2 e^{-2\lambda|\tau|}$. Determine the power spectral density of the process.

10. The power spectral density function of a wide–sense stationary process is given by $S(W) = \begin{cases} 1; & |W| < W_0 \\ 0; & \text{otherwise} \end{cases}$. Find the auto–correlation function of the process.

PART B — (5 × 16 = 80 marks)

11. (i) If X and Y are independent Poisson random variables, show that the conditional distribution of X given $X + Y$ is a binomial distribution.

(ii) A random variable X is exponentially distributed with parameter 1. Use Tchebycheff's inequality to show that $P(-1 \le X \le 3) \ge \frac{3}{4}$. Also find the actual probability.

12. (a) (i) If the joint p.d.f. of two random variables X and Y is given by

$$f_{XY}(x, y) = \begin{cases} x + y; & 0 \le x, y \le 1 \\ 0; & \text{otherwise} \end{cases}$$

Find the p.d.f. of XY.

(ii) If $X_1, X_2, \ldots\ldots\ldots, X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P(120 < S_n < 160)$ where $S_n = X_1 + X_2 + \ldots\ldots + X_n$; and $n = 75$.

Or

(b) (i) If X, Y and Z are uncorrelated random variables with zero mean and standard deviation 5, 12 and 9 respectively, and if $U = X + Y$ and $V = Y + Z$, find the correlation coefficient between U and V.

(ii) If X and Y are independent exponential distributions with parameter 1, find the p.d.f of $U = X - Y$.

13. (a) (i) State the conditions under which the Poisson distribution is a limiting case of the Binomial distribution and show that under these conditions the Binomial distribution is approximated by the Poisson distribution.

(ii) Check whether the two random processes given by $X(t) = A\cos\omega_0 t + B\sin\omega_0 t$ and $Y(t) = B\cos\omega_0 t - A\sin\omega_0 t$ where A and B are uncorrelated, zero mean and equal variance random variables are wide sense stationary.

Or

(b) (i) Show that X is a discrete random variable taking positive integer values with memoryless property, then X is the geometric distribution.

(ii) If the wide sense stationary process $X = \{X(t)\}$ is given by $X(t) = 10\cos(100t + \theta)$ where θ is uniformly distributed in $(-\pi, \pi)$, prove that X is ergodic with respect to the auto–correlation function.

14. (a) (i) For the jointly wide sense processes X and Y prove that

$$|R_{XY}(\tau)| \le \sqrt{R_{XX}(0)\,R_{YY}(0)} \le \frac{1}{2}\left(R_{XX}(0) + R_{YY}(0)\right).$$

(ii) For an input–output linear system $(X(t), h(t), Y(t))$, show that

(1) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$

(2) $R_{YY}(\tau) = R_{XY}(\tau) * h(\tau)$.

Or

(b) (i) Show that the inter–arrival time of a Poisson process with intensity λ obeys an exponential law.

(ii) The power spectral density of a zero mean wide sense stationary process $X(t)$ is given by $S(W) = \begin{cases} k; & |w| < w_0 \\ 0; & \text{otherwise} \end{cases}$ where k is a constant. Show that $X(t)$ and $X\left(t + \frac{\pi}{W_0}\right)$ are uncorrelated.

15. (a) (i) The power spectral density of a wide sense stationary process is given by $S(W) = \begin{cases} \frac{b}{a}(a - |w|); & |w| \le a \\ 0 & ; |w| > a \end{cases}$. Find the auto–correlation function of the process.

(ii) The power spectrum of a wide sense stationary process $X = \{X(t)\}$ is given by $S(W) = \frac{1}{(1 + w^2)^2}$. Find the auto–correlation function and average power of the process.

Or

(b) (i) Show that for an input–output system $(X(t), h(t), Y(t))$; $S_{YY}(W) = S_{XX}(W)|H(W)|^2$ where $H(W)$ is the system transfer function, and the input X is wide sense stationary.

(ii) Find the power spectral density of a wide sense stationary process with auto–correlation function $R(\tau) = e^{-\alpha\tau^2}$ where α is a constant.